

Spinodal decomposition during the hadronization stage?

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The central goal of high-energy nuclear collisions is to explore the expected phase transition from the familiar hadronic world to a plasma of quarks and gluons. This phase change of strongly interacting matter is expected to be of first order with a critical temperature T_c in the range of 150-200 MeV, although this remains to be determined experimentally. The starting point for the present study [1] is the observation that if the phase transition is of first order, then the hadronization should proceed by spinodal phase separation, a phenomenon well known from other areas of physics, including nuclear multifragmentation where it gives rise to highly non-statistical fragment size distributions with a preference for equal masses[2].

A first-order phase transition occurs when the appropriate thermodynamic potential exhibits a convex anomaly. There is then an interval of energy density throughout which the pressure has a negative derivative, $\partial p / \partial \epsilon < 0$. This anomalous behavior identifies the region of spinodal instability where small deviations from uniformity are amplified, as the system seeks to break its uniformity and separate into the two coexisting phases consistent with the given ϵ .

Employing a simple cubic spline function to approximate the equation of state $p(\epsilon)$ between the hadron gas and a bag of quarks and gluons, we have used relativistic fluid dynamics to estimate the growth rates γ_k for the unstable modes in the spinodal region. To include finite-range effects we refine the usual local density approximation $p(r) = p(\epsilon(r))$ by convoluting $p(r)$ with a kernel $g(r)$ of specified range a . This dispersion relation is then augmented by the Fourier transform $g_k, \gamma_k^2 = -(\partial p_0 / \partial \epsilon_0) k^2 g_k$, thereby ensuring that the characteristic fluid-dynamical linear growth for small k evolves into a rapid drop-off at large k . The corresponding maximum growth rate, $\gamma(k_0)$, identifies the most rapidly amplified wave length $\lambda_0 = 2\pi/k_0$ which will therefore become dominant in the emerging pattern of plasma blobs.

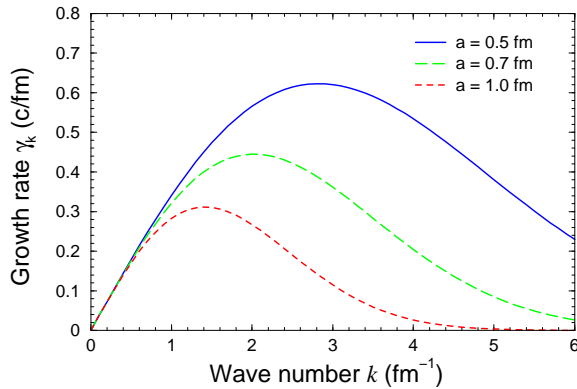


FIG. 1: The spinodal dispersion relation γ_k at the energy density $\tilde{\epsilon}$ where the pressure $p(\epsilon)$ decreases most rapidly, $\partial^2 p / \partial \epsilon^2 = 0$, for three values of the smearing range a (from Ref. [1]).

As the system expands and the energy density traverses the spinodal region, the growth rate $\gamma_k(\epsilon(t))$ of a given mode k increases from zero, exhibits a maximum at $\tilde{\epsilon}$, and then reverts to zero. The total growth factor is then approximately equal to $G_k = \exp(\Gamma_k)$ with the amplification coefficient Γ_k being

$$\Gamma_k \equiv \int_{t_B}^{t_A} \gamma_k(\epsilon(t)) dt \approx \gamma_k(\tilde{\epsilon}) \Delta t_{\text{eff}}, \quad (1)$$

where the effective duration of the instability is $\Delta t_{\text{eff}} \approx (\pi/4)(t_B - t_A)$ in the cubic spline approximation to $p(\epsilon)$.

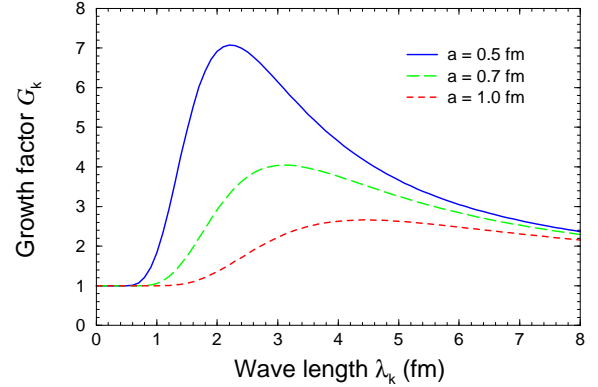


FIG. 2: The factor $G_k = \exp(\Gamma_k)$ by which the amplitude of undulations having wave length $\lambda_k = 2\pi/k$ grows during the expansion through the spinodal region of energy density ϵ (from Ref. [1]).

Spinodal decomposition is intimately linked to the occurrence of a first-order phase transition and is important in many areas of physics as a mechanism for pattern formation. The present study is focussed on high-energy nuclear collisions and we have investigated the importance of spinodal decomposition while the expanding matter passes through the phase coexistence region. Working within the framework of relativistic fluid dynamics and basing our analysis on commonly employed assumptions about the equation of state (supplemented with a simple spline procedure), we have found that a significant degree of amplification may occur while the matter passes through the mechanically unstable region of phase coexistence. In view of the potential utility of spinodal decomposition as a diagnostic tool, it seems worthwhile to pursue this issue with dynamical treatments that take better account of the spatio-temporal features of the system.

[1] Jørgen Randrup, Phys. Rev. Lett. **92**, 122301 (2004).

[2] Philippe Chomaz, Maria Colonna, and Jørgen Randrup, Physics Reports **389**, 263 (2001).